Mathematics Components of Entrance Exams at Peruvian State Universities: Rigorous Selection Criteria for Higher Education in an Emerging Economy

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Introduction

Since 1995, I have spent the northern hemisphere's summers serving as a volunteer mathematics instructor for engineering majors at the *Facultad de Tecnología* of the *Universidad de San Francisco Xavier* (USFX), the state university in Sucre, Bolivia. I have often been impressed with the backgrounds, capabilities, and achievement levels of most of my students, and in particular of those who identified themselves as natives of Perú. The Peruvian students were invariably among the best performers on exams, and shared with me that they had chosen to attend a Bolivian institution because they were unable to pass the rigorous entrance exams of state universities in their own country, in spite of having spent one or two years attending private academies after having completed high school in order to prepare for such tests. I was told that Peruvian state universities generally have very low admissions rates. These observations were repeatedly confirmed during my frequent cross-border excursions into Perú while classes at the USFX were interrupted by strikes, national holidays, or academic recesses.

Decades ago, the Bolivian government decided to "democratize" state universities by making them generally accessible to large segments of the population and admitting a less well prepared student body than was common in other South American nations. The state universities in the cities of La Paz and Cochabamba have maintained somewhat stringent admissions criteria, but other state universities, such as the USFX, typically admit about half of all applicants, most of whom are able to begin their studies immediately after having finished high school. For students who are unable to pass the entrance exam for engineering majors of the USFX's *Facultad de Tecnología* , the school offers open-admissions *carreras técnicas* (technician programs) similar to vocational programs at community colleges in the U.S. These academic tracks include three-year courses of study in information technology, food science, industrial chemistry, and interior design leading to the *técnico superior* (senior technician) diploma, as well as two-year *técnico* (technician) degree programs which train future electricians and auto mechanics. Most students who complete the *carreras técnicas* leave the university and attempt to enter the work force, although a smaller number are able to transfer some of their credits toward undergraduate engineering programs. Whereas a relatively low percentage of students admitted to the *Facultad de Tecnología's* engineering programs succeed in obtaining their diplomas within six years, the degree completion rate is high in most other disciplines. A large number of graduates from public institutions flood the Bolivian job market each year, and few are able to secure employment related to their studies, in part because the less stringent admissions practices of Bolivian state universities do not confer a level of prestige comparable to that of the much more selective Peruvian schools.

I hope that this article will alert American readers of wide disparities in selection criteria for admission to higher education which exist throughout Latin America, and inform them that in many developing countries, access to a state-subsidized education at a public university is limited to only the best prepared and most highly motivated, and often to students of privileged socioeconomic origins.

The Entrance Exams

Intrigued by reports of difficult entrance exams, I travelled to Perú late in the summer of 2010 after having completed my teaching services at the USFX. During this visit, I investigated university admissions practices, and was able to obtain copies of old entrance exams of various public and private universities from kiosks on the campuses of these institutions. Such copies are routinely distributed to prospective students in order to allow them to prepare for the entrance exams.

Nine representative mathematics questions selected from the exams of public universities are listed in Appendix A, along with suggested solutions. These nine questions were drawn from entrance exams of the five universities listed below along with their 2004 admissions rates (Díaz, "Educación superior en", 93).

- *Universidad Nacional del Callao* (UNC: 22%), a state university located in the northwestern outskirts of Lima.
- *Universidad Nacional Pedro Ruiz Gallo* (UNPRG: 20%), the state university in the northern city of Chiclayo.
- *Universidad Nacional de Trujillo* (UNT: 31%), the state university in the northern city of Trujillo.
- *Universidad Nacional de Ingeniería*, also known as the *Uni* (16%). This state institution, located in Lima, is Perú's premier university for engineering studies. It enjoys a level of prestige similar to those held by the *École Polytechnique* in France or the Indian Institute of Technology in India. Although the *Uni*'s 2004 admissions rate was not the nation's lowest, it should be noted that only the most talented students even consider preparing for this university's exam, leaving little doubt that this institution is the nation's most selective. The problems from the *Uni*'s entrance exam are by far the most difficult ones listed in Appendix A.

- *Universidad Nacional Mayor de San Marcos* (UNMSM: 8%), a prestigious state university located in Lima and reportedly the oldest university in the western hemisphere. The UNMSM is Perú's most prestigious comprehensive university and has its lowest admissions rate.

Readers familiar with the standard high school mathematics curriculum in the U.S. will recognize that all of the problems listed in Appendix A are challenging. I have posed most of these problems to students enrolled in several of my undergraduate mathematics classes at my U.S. institution, and found that even third year mathematics majors were often unable to respond correctly to many questions. In some instances, the problems require a familiarity with topics such as discrete structures and linear algebra that are usually only taught to second year mathematics majors enrolled at American universities. The exam questions' level of difficulty surpasses that of virtually all of the problems listed in commonly used American pre-calculus and calculus textbooks (Stewart, Larson et al, Purcell, Stein, Tierney, and Thomas et al). In addition to familiarity with background material, the Peruvian entrance exam problems require insight, pattern recognition capabilities, and the ability to carry out clever manipulations that would typically be acquired only after extensive preparation at private academies and secondary schools.

I also read an entrance exam of the *Universidad Nacional Frederico Villareal,* a state university located in Lima, and found the mathematics questions to be comparable in nature, scope, and difficulty to those which appeared on exams of other state universities. The entrance exams of state universities also contained challenging questions related to the physical and life sciences, geography, Spanish language, Latin American literature, as well as world and Peruvian history.

The two examples below (problems A6 and B7) graphically illustrate how markedly more challenging public institutions' mathematics entrance exam questions are, than those of private universities.

Problem A6: Let $A = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Then the value of $A^{42} + A^{55}$ is (a) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ (e) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Problem B7: Adriana was asked "*how much is 125 % of 120?*. The correct response is (a) 180 (b) 150 (c) 125 (d) 15

Problem A6 appeared on a recent entrance exam of the public *Universidad Nacional de Ingeniería* (*Uni*). Experimenting with raising the matrix A to various powers, one finds that

$$A^{2n} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and that $A^{2n+1}=A$, so that upon evaluating $A^{42} + A^{55}$, one obtains the answer (b). While this would probably be a reasonable test question for a sophomore linear algebra class at my U.S. institution, I find it impressive that students would be expected to demonstrate the insight and pattern recognition capabilities required to solve such a problem in order to be admitted to an undergraduate institution.

Problem B7 was recently posed to students applying to the *Universidad Inca Garcílasco de la Vega* (UIGV). By direct proportions, the answer is (b) 150.

More extensive lists of sample entrance exam questions (with suggested solutions) of public and private institutions are provided in

Appendix A and Appendix B, respectively. Readers will recognize that the problems of Appendix A require insight, cleverness, and background uncommon even among sophomore mathematics majors in the U.S., whereas those in Appendix B usually require little more than a sequence of trivial computations and symbol manipulations. Ten representative mathematics problems drawn from the exams of private institutions are listed in Appendix B, along with suggested solutions. The three universities from which the exam questions were obtained are listed below along with their 2004 admissions rates (Díaz, "Educación superior en", 93).

- The *Universidad Particular de San Martín de Porres* (UPSMP: 90%). This is a private institution with two campuses in the metropolitan Lima region and another one in the northern city of Chiclayo.
- The *Universidad Inca Garcílasco de la Vega* (UIGV: 97%). This private institution has one of the highest admissions rates in the nation. I was told informally that its tuition is among the lowest of private universities and that many of the students are from lower middle class and working class urban backgrounds. When traveling along the Pan American highway, I noticed that the UIGV had a large campus in the slums of the northern outskirts of Lima, an indication that the student body is largely underprivileged.
- The *Universidad Tecnológica del Perú* (UTP: 85%). The UTP is a consortium of private universities focusing on engineering and computer science. Throughout my travels, I became aware that the UTP has campuses in all of Perú's main cities, including Arequipa, Chiclayo, Trujillo, and the nation's capital. The main campus is in the San Isisdro upper middle class neighborhood of Lima, leading me to believe that much of the student body is of more affluent backgrounds than would be common at other private universities. The UTP also maintains a system of branch campus community colleges known as IDAT throughout all of Perú.

An examination of the questions listed in Appendix B confirms that they are much less challenging than those of Appendix A. In the case of the UIVG, the questions are trivial, making it clear that for this institution, the "entrance exam" is little more than a formality and that the only entrance requirement is the ability to pay the school's tuition. The questions on the exams of the UPSMP and UIGV would be recognized by most readers as "easy". I found these problems to be generally comparable in difficulty to those of other private schools with admissions rates of 90 % or higher.

The most challenging mathematics questions of any private university were to be found on the exams of the UTP, as these required moderate levels of cleverness as well as familiarity with exponential, logarithmic, and trigonometric functions, aptitudes, backgrounds and skills which would probably not be easily accessible to students who had attended public high schools in working class neighborhoods. The UTP's exam questions were, nevertheless, much less challenging than those listed in Appendix A. I was unable to obtain copies of old entrance exams of the *Pontificia Universidad Católica del Perú*, the nation's most prestigious, selective and expensive private university.

Admissions Practices of Peruvian Universities

Public institutions are by far the most selective ones within Perú's higher education system. A study of the admissions practices of all nationally accredited Peruvian universities indicated that in 2004, the overwhelming majority of public institutions had admissions rates of 25 % or less, while the majority of private ones had rates of 77 % or higher (Díaz, "Educación superior en", 93). The same study revealed that 26 private universities had admissions rates above 80 %, and twelve had rates of at least 90 %. The nineteen schools with the lowest admissions rates were all public, and only in twentieth place did one find a private university -the *Pontificia Universidad Católica del Perú* (Díaz, "Educación superior en", 92). Another report indicated that from 1968 to 2000, the number of applicants per opening rose from a ratio of 3:1 to more than 6:1 among state universities, whereas the ratio fell from 4:1 to approximately 3:2 among all private institutions between 1987 and 2000 (Soto Nadal, "Diagnóstico de la", 56). It should be noted that while low, the admissions rates of state universities understate the level of selectiveness of such institutions, given that many of the students who pass do so after having previously attempted (and failed) the exams for several consecutive semesters.

Socioeconomic Implications of Stringent Admissions Practices

Peruvians reported to me that for over a century, their nation's public universities have maintained selective admissions practices in part to restrict the entry of a large number of young people into an anemic job market. Like most developing countries, Perú is demographically a very young nation; a relatively high birth rate of 22 per 1,000 residents had contributed to an annual population growth rate of 1.9 % throughout the period 1985-2000. By 2001, the percentage of the population between the ages of 18 and 25 participating in post-secondary education had risen to 32 %, creating a huge demand for higher education which could not be met by public institutions alone (Oppenheimer, 177).

The proliferation of private universities throughout Perú is largely a response to a demographic surge of young adults aspiring to enter the nation's higher education system. Between 1970 and 2005, the number of Peruvians between the ages of 17 and 20 rose from 1.5 million to 3.2 million (Díaz, "Educación superior en", 87). During this time, public universities, became even more selective, reducing their admissions rates from 31% in 1970 to 18 % in 2004 (Díaz, "Educación superior en", 87). This generated a massive backlog of young people who could not be incorporated into the public higher education system. Private institutions were quick to respond to this need; of the 26 new universities created between 1995 and 2005, 19 were private (Díaz, "Educación superior en", 90).

Perú has boasted one of the fastest rates of economic growth in Latin America during the past two decades (Oppenheimer, 190). Nevertheless, much of this expansion has been made possible by surging exports in mining, textiles and agriculture rather than through value-added products (Lizarzaburu, "Perú alberga la"). Consequently, few employment opportunities for mathematically trained university graduates have been created in the past few decades, circumstances which the central government in Lima has used to justify continuing to limit the number of openings at state universities.

A pattern of macroeconomic growth emerged in Perú when Alberto Fujimori took office in 1990. He ushered in a wave of market-oriented reforms and policies that have been continued by his two democratically elected successors. Growing demand for commodities from rapidly expanding economies in Asia helped to sustain the bonanza during the past decade. The abrupt loss by of tariff exemptions by Bolivia in 2008 on its exports of textiles and furniture to the U.S. resulted in the relocation of many factories to Perú, further bolstering the Peruvian economy (Savoye, "Class Struggle, Regional"). These and other factors resulted in Perú's economy growing more rapidly than that of any other nation in the region in recent times ("El FMI ratifica").

Nevertheless, Peruvian society continued to endure the consequences of alarming levels of wealth and income inequality, as a large portion of Perú's population has not benefitted from the economic growth of the past two decades (Schuldt, "Desigualdad Económica y"). Most of the economic growth has been concentrated in the metropolitan Lima region and along the northern coast; a recent survey revealed that during the past four years, personal income among Lima households had risen by 27 % in the Peruvian currency, which had, in turn, appreciated against the dollar and other western currencies during the same period ("Ingresos en Lima"). A resurgence of mining activity has also benefited certain highland cities such as Arequipa and Cajamarca. Other regions, such as the northeastern jungle surrounding the city of Bagua and the southeastern *altiplano* region bordering Bolivia and Chile have continued to languish in extreme poverty and isolation.

Even in Perú's political and economic capital, large segments of the population seemed to be unaffected by the wave of unprecedented prosperity that was sweeping through other sectors of Peruvian society. Travelling by bus through the northern and eastern outskirts of Lima, I was often astounded by the disturbingly large number of people living in *pueblos jóvenes* (young towns), shanty towns constructed in precarious locations. These crime-ridden slums were built in the 1980's and early 1990's by peasants fleeing from terror inflicted by the Shining Path guerrilla organization throughout the southern and central highlands (Strong). The *pueblos jóvenes* rarely offer even basic services such as sewage or running water to their residents, who often have to commute to the wealthy southern suburbs of Lima in order to subsist as part of *la economía informal*, peddling food, drinks, and contraband goods to their more prosperous countrymen.

The composition of the student body resulting from stringent criteria for admission to state universities appears to be both a symptom and a cause of the sharp socioeconomic disparities which pervade Peruvian society. The majority of students enrolled at such institutions are from upper middle class backgrounds, and have attended private schools and preparatory academies, where they acquired the academic background and rigorous coursework necessary to pass the entrance exams. Such preparatory academies have proliferated throughout Perú in response to the doubling of the number of applicants per vacancy between 1968 and 2000. Attending such institutions after regular school hours and after having completed high school is seen as essential to acquiring even a modest chance of passing state universities' entrance exams. Private academies typically charge monthly tuition of between 150 and 300 soles (US $ 55-110), a prohibitively high cost to families of moderate incomes ("Centros Preuniversitarios"). Many Peruvian households would have difficulties in meeting such expenses; civil servants such as teachers earn monthly incomes of about 1,100 soles (US $ 407) (Quispe, "Gobierno fija sueldo"). It should be noted that the cost of attending academies does not include tuition paid to private high schools. Additionally, enrolling in academies requires significant commitments of time, a luxury often unavailable to less wealthy students, who face enormous financial pressures to work while still in high school. Given that significant sectors of the population have not benefited from recent patterns of economic growth and that high levels of income and wealth inequality persist, it is evident that that adolescents and young adults from less favored socioeconomic origins are unlikely to be able to attend private academies, and are thus disadvantaged in preparing for the entrance exams of state universities.

Less privileged peers who graduate from public high schools often find that private universities or public ones in Bolivia are their only options for competing in a saturated labor market. Mainly students from affluent backgrounds are able to enjoy the benefits of a state-subsidized education and the prestige that accompanies diplomas from public institutions. Graduates of less selective private universities, who often are of less privileged backgrounds than their peers at state institutions, have greater difficulties in securing employment, and in this manner, different socioeconomic origins from childhood and early adulthood are perpetuated into later stages of life. Other youngsters fare even worse, as the large number of Peruvian youth who are born into extreme poverty and have endured child labor and other hardships rarely succeed in completing high school; in 2002, 31 % of all adolescents were not matriculated in secondary schools (Oppenheimer, 178). Recent investigations revealed that indigent Peruvian children and adolescents were being kidnapped and sold into human trafficking rings which subjected them to slave labor, prostitution, extortion, and murder on a widespread basis (Daniels, "Allí nadie se").

In addition to not benefiting from the prestige associated with degrees awarded by state universities and to being hampered by the financial burden of having to pay tuition, the often underprivileged students who attend private universities are generally exposed to educational experiences of much lesser quality. Private universities are usually staffed by part-time adjunct faculty who work in several institutions and have neither the time nor the motivation to engage in research. At the UPSMP, for example, 85.4 % of all faculty were adjunct in 2001. Students matriculated at such institutions are deprived of the opportunity to study under professors who are up to date with current developments in their fields and can transmit the intellectual curiosity and research skills required to thoroughly investigate a problem. Additionally, the quality of instruction suffers greatly at private universities due to the demands placed on professors who must work at several institutions. In contrast, the majority of faculty at state institutions are full-time tenured or tenure-track employees, and many are active researchers, in this manner providing a greatly enriched academic environment for the students (Soto Nadal, "Diagnóstico de la", 26-34).

The level of difficulty of the mathematics questions on entrance exams of Peruvian state universities results in a greatly reduced level of representation of students from less favored socioeconomic backgrounds. Implementing strategies to overcome such obstacles to upward mobility will be essential if the Andean nation is to succeed in distributing the benefits of its recent pattern of rapid economic growth among all sectors of society.

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Appendix A
Selected Problems from mathematics Portions of Entrance Exams of Various State Universities

Various representative problems from the entrance exams are listed below. I have translated the questions into English. Question 1 was posed by the UNT.

1. If α, β, and θ are acute angles such that

$$\frac{\alpha}{3}=\frac{\beta}{4}=\frac{\theta}{5} \qquad (1)$$

and

$$\tan(\alpha+\beta)=\cot(\theta) , \qquad (2)$$

then the value of $4\alpha+\beta-2\theta$ in radians is (a) π/2 (b) π/3 (c) π/4 (d) π/8 (e) π/5.

Question 2 appeared on a recent entrance exam of the UNPRG.

2. If

$$5^{n}+5^{n+1}+5^{n+2}+5^{n+3}+5^{n+4}=780, \qquad (3)$$

then the value of n is

(a) 1 (b) 0 (c) 2 (d) 5 (e) 3 .

Questions 3 and 4 appeared on a recent entrance exam of the UNC directed toward accounting, business administration, and nursing majors. (The one for science, mathematics, and engineering majors contained more challenging problems).

3. Denoting the square root of -1 by i, then if N is a positive integer such that $i^N = -1$, then the largest possible remainder upon division of N by 8 is (a) 7 (b) 5 (c) 4 (d) 3 (e) 6.
4. If $\tan\alpha - \cot\alpha = 3/2$, then the value of $\sec^2\alpha + \csc^2\alpha + 3/4$ is (a) 8 (b) 7 (c) 9 (d) 6 (e) 5 .

Question 5 appeared on a recent entrance exam of the UNMSM.

5. Let $x - x^{-1} = 1$ $(x \neq 0)$. (5)
Then the values of $x^2 + x^{-2}$ and $x^3 - x^{-3}$ are (a) 2 and 3 (b) 2 and ½ (c) 3 and 1/3 (d) 3 and 4 (e) 4 and ¼ .

Questions 6- 9 appeared on a recent entrance exam of the *Uni*.

6. Let $A = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Then the value of $A^{42} + A^{55}$ is (a) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$
(b) $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ (e) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$
7. Given the functions f={(3,1), (2,-3), (5,0), (4,-4), (1,1)}, g={(-4,3), (-2,7), (0,0), (1,5), (2,1)} and h={(1, -4), (3,-2), (5,0), (7,2)}, the composition of functions $f \circ g \circ h$ is (a) {(1,0), (5,1)} (b) {(3,-3), (5,-4)} (c) {1,1), (7,1)} (d) {(1,1), (2,-3)} (e) {(3,-1), (7,1)}.

8. In a lottery, 6 chips are randomly selected without replacement from a collection of 20 chips, each of which is numbered from 1 to 20. There are no duplicates in the latter. Find the probability that the sum of the numbers on the first two chips selected attains the value 30.
(a) $\frac{1}{190}$ (b) $\frac{1}{100}$ (c) $\frac{1}{80}$ (d) $\frac{1}{40}$ (e) $\frac{1}{38}$
9. Consider the sequence defined as follows: $a_1 = 0$, $a_2 = 1$, $a_3 = \frac{1}{2}$, $a_4 = \frac{3}{4}$, $a_5 = \frac{5}{8}$, $a_6 = \frac{11}{16}$, $a_7 = \frac{21}{32}$, $a_8 = \frac{43}{64}$, ...
The sequence $\{a_n\}$ converges to the value (a) $\frac{7}{12}$ (b) $\frac{5}{8}$ (c) $\frac{2}{3}$ (d) 1 (e) $+\infty$.

Solutions and Observations

1. It follows from (2) that $\alpha+\beta = \frac{\pi}{2} - \theta$. Applying (1), we deduce that the correct answer is (c). It is noteworthy that the students were asked to evaluate a more complicated expres sion such as $4\alpha+\beta-2\theta$ instead of a simpler term such as θ so as to make it impossible for them to arrive by elimination at a correct answer by merely substituting five possible answers into equations (1) and (2). I suspect that the problem would be viewed as very challenging by most students enrolled in high school trigonometry courses in the U.S.
2. This is a rare instance in which the correct answer (b) could be obtained by elimination,

noting that for $n=1$, the last term (5^{n+4}) of the left side of (3) attains the value 3,125, which exceeds 780, leaving $n=0$ as the only possible choice. Alternatively, students could factor the left side into $5^n(5+25+125+625)$ and note that the expression in parentheses adds to 780. When I presented this problem to my second semester calculus class, only three of fourteen students were able to respond correctly, in spite of having spent several classes discussing finite sums and infinite series.

3. This problem requires students to experiment with various choices of N in order to conclude that the remainder upon division by 8 will always be 2 or 6, so the correct answer is (e). I view this question as being rather challenging for its intended audience, given that accounting and nursing majors at my U.S. institution often report never having encountered complex numbers.
4. Squaring both sides of the equation $\tan\alpha - \cot\alpha = 3/2$ and observing that $\tan\alpha = 1 / \cot\alpha$, one deduces that $\tan^2\alpha - 2 + \cot^2\alpha = 9/4$. From the Pythagorean identities, one obtains the relation $\sec^2\alpha + \csc^2\alpha = 2 \tan^2\alpha + \cot^2\alpha$, yielding the equation $\sec^2\alpha + \csc^2\alpha + ¾ = 7$, from which we conclude that the correct answer is (c).
5. Squaring both sides of (5), one deduces that $x^2 -2 + 1/x^2 = 1$. Rearranging, one finds that

   $$x^2 + 1/x^2 = 3.$$

   Cubing both sides of (4) and expanding the left side yields

   $$x^3 - 3x + 3/x - 1/x^3 = 1.$$

   Equivalently,

   $$x^3 - 1/x^3 - 3(x - 1/x) = 1.$$

   Substituting (4) into the expression in parentheses, one finds that $x^3 - 1/x^3 = 4$, so that the correct answer is (d). The solution of this problem requires a familiarity with binomial expansions and a facility for algebraic manipulations that are probably attained only by better prepared U.S. high school seniors.
6. Experimenting with raising the matrix A to various powers, one finds that $A^{2n} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

   and that $A^{2n+1}=A$, so that upon evaluating $A^{42} + A^{55}$, one obtains the answer (b). While this would probably be a reasonable test question for a sophomore linear algebra class at my U.S. institution, I find it impressive that students would be expected to demonstrate the insight and pattern recognition capabilities required to solve such a problem in order to be admitted to an undergraduate institution.
7. Following the definition of the composition of functions, one arrives at the answer (c). Although this problem would be viewed as a "routine" one by students enrolled in the sophomore discrete structures course at my U.S. institution, the presence of such a problem on the *Uni*'s entrance exam provides an indication of the level of breadth and depth that prospective students are expected to attain prior to enrolling.
8. Identifying the five outcomes which satisfy the given condition, one finds that the probability is determined by the ratio $\dfrac{5}{\binom{20}{2}}$,

   from which one obtains the answer (e). I assigned this problem for homework to my freshman finite mathematics class consisting mainly of music and liberal arts majors, and found that only four of 29 students were able to respond correctly.

9. I presented this problem to a second-semester calculus class and to an upper division class devoted to partial differential equations and boundary value problems. Not one student had the pattern-recognition capabilities to determine that the nth term of the sequence is given by

$$a_n = \frac{2^{n-1} + (-1)^{n-2}}{3 \cdot 2^{n-2}} \qquad (6).$$

Nevertheless, when presented with this relation, the majority of students in both classes were able to evaluate $\lim_{n \to \infty} a_n$, proceeding by first dividing the numerator and denominator by 2^{n-1} and then allowing n to approach infinity:

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{1 + \frac{-1}{2^{n-1}}}{\frac{3}{2}} = 2/3 \quad ,$$

in this manner obtaining the correct answer of (c). Obtaining the result (6) would be considered to be an unreasonably difficult freshman calculus problem at my U.S. institution. That this should be a representative question on an entrance exam of the *Uni* indicates that Perú's premier engineering institution is very selective.

Appendix B
Selected Problems from mathematics Portions of Entrance Exams of Various Private Universities

Questions 1-3 appeared on entrance exams of the *Universidad Particular de San Martín de Porres* (UPSMP).

1. After simplification, the value of the expression is $(x + 1)(x - 1) + (2 - x)(2 + x)$ is
   (a) 4 (b) 3 (c) 2 (d) x^2 (e) $-x^2$
2. If $\alpha - b = \sqrt{3} - 1$ and $\alpha - b = \sqrt{3}$, then the $\alpha^2 + b^2$ value of is
   (a) 6 (b) 5 (c) 4 (d) 3 (e) 2
3. The next term of the sequence {6 , 5 , 3 , -1 , -9 , ….. } is
   (a) -15 (b) -18 (c) -25 (d) -30 (e) -28

Questions 4-7 appeared on a recent entrance exam of the *Universidad Inca Garcílasco de la Vega* (UIGV), whose questions were by far the least challenging of all institutions examined, as their solutions required only very elementary arithmetic and algebra.

4. If Mario is able to walk 60 meters while Rodrigo is able to walk 40 meters, how many meters will Mario be able to walk while Rodrigo walks 60 meters?
   (a) 180 (b) 120 (c) 100 (d) 90 (e) 80
5. The sales price of an item was lowered by 20 %. By what percentage must the price next be raised in order to bring it back to the original level?
6. Álvaro took half of a watermelon and distributed it into equal parts among eight children. What fraction of a watermelon did each child receive? (a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{32}$ (d) $\frac{8}{10}$ (e) $\frac{3}{8}$
7. Adriana was asked "*how much is 125 % of 120?*. The correct response is (a) 180 (b) 150

(c) 125 (d) 15

Questions 8-10 appeared on a recent entrance exam of the *Universidad Tecnológica del Perú* (UTP).

8. An urn contains 99 chips which are numbered 1 through 99. If chips are randomly removed from the urn without replacement, what is the minimum number of chips that must be removed in order to be certain of having removed nine odd-numbered chips?
   (a) 59 (b) 58 (c) 57 (d) 56 (e) 60
9. If the difference between the roots of the polynomial is 1, then the $p(x) = x^2 - 7x - m$ is 1, then the value of $m^2 + 12m + 1$ is (a) 2 (b) 0 (c) 1 (d) 3 (e) 4
10. If $x^{\sqrt{x-6}} = ({}^{\sqrt{3}}\sqrt{x})^{x-6}$, then the product of the values of which satisfy this equation is
   (a) 42 (b) 54 (c) 48 (d) 36 (e) 27

Solutions and Observations

1. This is a trivial algebraic simplification exercise that requires little more than familiarity with the "difference of two squares" result commonly taught in the eighth grade in the U.S.: $(x + 1)(x - 1) + (2 - x)(2 + x) = x^2 - 1 + 4 - x^2$, which reduces to 3. So the correct answer is (b).
2. This too is a simple exercise that tests little more than a student's ability to engage in symbol manipulation at a level commonly taught in elementary high school algebra courses: $a^2 + b^2 = (a - b)^2 + 2ab = \sqrt{3} - 1)^2 + 2\sqrt{3} = (4 - 2\sqrt{3}) + 2\sqrt{3} = 4$, so the correct answer is (c).
3. By inspection, the n^{th} term in the sequence is $7 - 2^{n-1}$. Hence, the sixth term is $7 - 2^5 = -25$, so the correct answer is (c). It should be noted that the pattern recognition skills required to solve this problem are much simpler than those of problem 9 from Appendix A, for example, an indication of how entrance exam questions are typically much easier for private universities than for public ones.
4. By direct proportions, the answer is (d) 90.
5. By direct proportions, the answer is (c) 25.
6. Carrying out the trivial computation, ½ ⅛, one obtains the value 1⁄16, so the correct answer is (a).
7. By direct proportions, the answer is (b) 150.
8. Given that 49 of the chips of the even-numbered (all of the even integers from 2 to 98), then under the worst-case scenario, if the first 49 chips were to be even, then we would be assured that the next 9 would be odd, as there would be no even-numbered ones left. Hence, the maximum number of selections is 49 + 9 = 58, so the correct answer is (b).
9. Denoting the two roots of $p(x)$ by x_1 and, x_2 and observing that the middle coefficient of $p(x)$ is -7, it follows that $x_1 + x_2 = 7$. Given that the difference between the two roots is given to attain the value 1, we obtain the system,
   $\begin{cases} x_1 + x_2 = 7 \\ x_1 + x_2 = 1 \end{cases}$, which can be solved (by elimination or by adding the two equations) to yield $x_1 = 4$ and $x_2 = 3$. We then determine that $m = - x_1 x_2 = -12$. Substituting this value for into the polynomial, $m2 + 12m + 1$, we obtain the value 1, so the correct answer is (c).
10. Using properties of exponents to simplify the right side, we obtain the equation, $x^{\sqrt{x-6}} = x^{\sqrt{x-6})/\sqrt{3}}$. Equating the exponents yields a quadratic equation in: $3(x - 6) + (x - 6)^2$, whose solutions are $x = 6$ and $x = 9$, whose product attains the value of 54, so the correct answer is (b). This question is clearly more challenging than those posed by the UPSMP and the UIGV, as assesses the ability to use properties of exponents in a fairly clever way. Additionally, by

asking test takers to determine the product of the solutions rather than the solutions themselves, the test designers make it impossible for students to substitute given values of x into the original equation and in this manner obtain the correct answer by elimination. Nevertheless, the level of difficulty still falls short of that generally found on entrance exams of state universities.